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Spacetime energy decreases under world-sheet RG flow

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Abstract: We study renormalization group flows in unitary two dimensional sigma models with asymptotically flat target spaces. Applying an infrared cutoff to the target space, we use the Zamolodchikov c-theorem to demonstrate that the target space ADM energy of the UV fixed point is greater than that of the IR fixed point: spacetime energy decreases under world-sheet RG flow. This result mirrors the well understood decrease of spacetime Bondi energy in the time evolution process of tachyon condensation.

Keywords: Renormalization Group, Bosonic Strings, Superstrings and Heterotic Strings, Tachyon Condensation.

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Contents

1. Introduction

Perturbative string theory establishes several intriguing connections between world-sheet and spacetime dynamics. As is well known, consistency of classical string propagation requires the world-sheet sigma model to be conformal, which in turn imposes α' corrected Einstein equations on spacetime fields [1]–[9]. The spectrum of fluctuations about solutions to the spacetime equations is in one-to-one correspondence with marginal operators of the corresponding world-sheet CFT, and the scattering of these fluctuation modes is computed by world-sheet correlators of the corresponding marginal operators. In this paper we will be concerned with yet another aspect of perturbative string dynamics, namely the connection between time evolution of the background and world-sheet renormalization group (RG) flows. The idea that these processes are related has been around for a long time, and has occasionally been employed in practical computations, most notably in the recent studies of localized closed string tachyon condensation [10]–[15] that have motivated the present work. Nonetheless the existing evidence for a definite relationship between these two processes appears rather scarce. Our aim in this paper is to present a non-trivial consistency check of the conjectured relationship between time evolution and world-sheet RG flow.

This conjectured relationship springs from the following observation: Consider a type-II string background of the form $\mathrm{CFT}_1 + \mathrm{CFT}_2$, where CFT_2 is any unitary conformal field theory with $\hat{c} = d$, and CFT_1 is the free sigma model on $R^{9-d,1}$ with fields X^a ($a = 0, \ldots, 9-d$). It is not difficult to see that the operator spectrum of CFT_2 determines the stability of this background under time evolution. In fact, a conformal operator O of dimension (δ, δ) in CFT_2 may be combined with a momentum factor from CFT_1 to yield a marginal operator in the full CFT: $\hat{O} = e^{iP\cdot X} O$ where $\alpha' P^2/2 + \delta = 1$. Thus the world-sheet operator $\hat{O}$ corresponds to a spacetime fluctuation with squared mass $M^2 = 2(\delta - 1)/\alpha'$ and so represents a tachyon or instability when O is a relevant operator. We therefore conclude that the existence of relevant operators in the spectrum of CFT_2 implies an instability of the corresponding string background.

It is tempting to go beyond such an 'infinitesimal' statement and to conjecture a relation between the full dynamical evolution in string theory and renormalization group flows on the world-sheet. Clearly, the two sides have many features in common. A world-sheet RG flow away from an unstable string background ends at an infrared conformal field theory that may generically be expected to be stable. The IR fixed point can be a theory with a mass gap, corresponding to a flow toward a noncritical string theory and will not be considered in this note. Similarly, after all the dust has settled, the dynamical process of tachyon condensation is generically expected to decay into a stable solution of string theory. It has often been conjectured that the endpoints of these two processes are the same (see e.g. [16, 17, 18]). In our attempt to provide evidence for such a conjecture, we shall mainly focus on a special class of string backgrounds for which CFT_2 is a sigma model onto an *asymptotically flat* d dimensional space. We also restrict attention to condensation processes throughout which spacetime remains asymptotically flat. This requires, in particular, that the relevant operators of CFT_2 are *localized* in target space. These assumptions will be relaxed slightly toward the end of the paper.

Intuitively, the decay of an unstable string background is a directional process: condensation processes are driven by the desire of a system to minimize its energy. Within the setup we specified in the previous paragraph, it is possible to make this intuition precise in terms of the so-called Bondi energy of asymptotically flat spacetimes, which can be considered as the difference between the total energy and the energy that has escaped away as radiation (see section 2.1 for a more precise formulation). Under appropriate assumptions it can be shown that the Bondi energy [19, 20] of a gravitational background decreases in time. In particular, if a gravitational background decays from a initial configuration toward a static solution, then the energy of the latter is smaller than the energy of the unstable solution in the far past. These mathematical results on Bondi energy feed the expectation that condensation processes in our string backgrounds proceed so as to lower some 'stringy energy.' Even though we do not know the precise definition of the latter, we expect it to agree with the relativists' notion of energy whenever the gravity approximation is valid.

This brings us to a description of our main results. Based on the arguments we have sketched above we shall take the directionality of stringy time evolution for granted, and show that the same behavior is found for the corresponding RG flows. More precisely, we will define some quantity on the world-sheet which decreases along the flows and relate it to gravitational energies.

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At first sight, this seems to be a rather simple task, since Zamolodchikov's famous c-theorem [21] asserts that the C function, an off-shell generalization of the central charge, is a (not necessarily strictly) decreasing function of RG scale. But for our non-compact backgrounds, the C function is completely determined by the asymptotics of target space and so remains constant along the flow. Nonetheless, by applying an infrared cutoff (compactification) to the target space, we will be able to employ Zamolodchikov's c-theorem and establish a directionality for RG flows in CFT_2. The cutoff in target space will allow us to remove the leading universal contribution from the C-function and to prepare a functional S that evolves from a higher value in the ultraviolet (UV) to a lower value at the infrared (IR) fixed point. We shall then show that the change in S in RG flow from the UV to the IR is precisely the difference between the ADM energies of the UV and IR target spaces. Thus, whenever quantities in the UV and IR limits can be well approximated by gravity computations, our results demonstrate that the ADM energy for the IR fixed point of our RG flows is lower than the ADM energy of the unstable UV fixed point. Using the known relation between ADM and Bondi energy (see also section 2.1 below), we therefore obtain a perfect agreement with the analogous result from the target space analysis that we have sketched above. Toward the end of this note we also present a generalization of our analysis to flows on non-compact sigma models with arbitrary but fixed asymptotics, and to flows on asymptotically conical two-dimensional sigma models that interpolate between spacetimes with different deficit angles.

We emphasize that CFT_1, which contains the time direction, is a mere spectator throughout the RG flow of CFT_2, whose unitarity justifies the use of the c-theorem. This may raise the following question in the mind of the reader: since the energy of a gravitational solution is roughly measured by the falloff near infinity of the time-time component of the metric, isn't the energy identically zero for all solutions we consider? The answer is that the physically relevant energy is determined by the Einstein frame metric,[1] not the string frame metric; thanks to the evolution of the dilaton, the time-time component of the Einstein frame metric will generically undergo quite non-trivial evolution under RG flow (although solutions such as non-dilatonic black holes, for example, would be excluded from our analysis). In the body of this paper, we will distinguish between the two metrics by denoting the string frame metric $G_{\mu\nu}$ and the Einstein frame one $g_{\mu\nu}$.

2. Essential background

The purpose of this section is to collect a number of well known results that provide much of the technical background for our analysis. In subsection 2.1 we recall the notion of Bondi energy in General Relativity, and we describe its monotonic decrease in light-cone time as well as its relation to the ADM energy. Subsection 2.2 contains a short review of the c-theorem. In subsection 2.3 we list some results from Tseytlin's computation [22] of the C function in α' perturbation theory.

[1] More precisely, due to the translational invariance in the $9-d$ spatial directions of CFT_1, it is the $d+1$ dimensional ADM energy that is properly defined (conceptually it is perhaps best to regard the other directions as compactified). Hence it is the $d+1$ dimensional Einstein frame metric we will employ.

2.1 Energy in asymptotically flat spacetimes

Relativists have defined at least two useful notions of energy in asymptotically flat spacetimes: ADM energy [23] and Bondi energy [19, 20]. These energies are both completely determined by the behavior of the spacetime metric 'at infinity'; they differ in *where* at infinity they are measured.

The ADM energy of a $d+1$ dimensional asymptotically flat spacetime is determined by the behavior of its metric on an infinite $d-1$ dimensional sphere i^0 that constitutes the boundary of space at any constant time. Explicitly, [2]

$$E_{\mathrm{ADM}} = \frac{1}{2\kappa^2} \int_{i^0} dS^j \left(\partial_i h_{ij} - \partial_j h_{ii} \right), \tag{2.1}$$

where $h_{ij} = g_{ij} - \delta_{ij}$ and the orientation of i^0 is chosen such that the normal vector points to the outside (i, j are spatial indices). It is not difficult to see that ADM energy is conserved under the gravitational time evolution.

Bondi energy, on the other hand, is determined by the behavior of the spacetime metric on a $d-1$ dimensional sphere of radius r and at time t (centered about the origin of space), where $r \to \infty$, $t \to \infty$ with $t - r = \lambda$ fixed. In other words, Bondi energy is evaluated at $\mathcal{I}^+$ on a Penrose diagram. The light-cone time λ labels where on $\mathcal{I}^+$ this energy is measured (see figure 1). An explicit formula for Bondi energy as an integral of a function of the metric over a particular sphere S^{d-1} on $\mathcal{I}^+$ may be found, for instance, in [24], but will not be needed thoughout here.

Note that Bondi energy is not conserved as a function of λ; this and other interesting features can be illustrated by the following thought experiment. Consider a universe that is empty except for a small bomb of mass M located at the origin of space. Let this bomb sit dormant until it explodes at $t = 0$. In this moment, all its energy is converted into radiation that propagates radially away to infinity. Since ADM energy is measured on infinite spheres at constant time which are never reached by the outgoing radiation, ADM

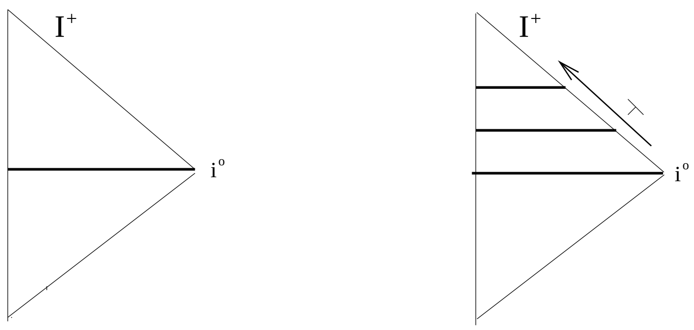

Figure 1: ADM energy, the energy contained in the largest spatial slice available in an asymptotically flat space (see the first Penrose diagram above) is measured by the fall-off of the metric at i^0. Bondi energy represents the energy contained in smaller spatial slices (see the second Penrose diagram above), measured by the fall-off of the metric at $\mathcal{I}^+$. Bondi energy is a decreasing function of λ.

[2]In our conventions the Einstein action is $S = \frac{1}{2\kappa^2} \int \sqrt{-g} R$.

energy of this spacetime is M, identical to that of a second universe in which the bomb never explodes. On the other hand, Bondi energy is determined on a family of light-cones whose apex is located in the origin of space. For $\lambda < 0$, the associated light-cone lies in a region of spacetime that never receives the news about the explosion. Hence, the Bondi energy at these values of λ is simply M. When $\lambda > 0$, on the other hand, the Bondi energy vanishes since it is measured on light-cones which lie entirely within flat space. Thus the Bondi energy of our spacetime is a monotonically decreasing function of λ which interpolates between the ADM energies of the 'initial' unstable solution (the bomb) and of the 'final' static solution (flat space).

The thought experiment of the previous paragraph illustrates several general properties of Bondi energy, established by relativists in the late 1970s (see the end of chapter 11 in [24] for a review and references). At $\lambda = -\infty$, the Bondi energy of a spacetime is equal to its ADM energy. In general, Bondi energy can also be shown to decrease in λ and the difference between the Bondi energy at two different light-cone times λ_1 and λ_2 is equal to the energy lost due to radiation in the band $[\lambda_1, \lambda_2] \subset \mathcal{I}^+$. Finally, if a given spacetime agrees with an auxiliary static spacetime in the region between two light-cone times λ_1 and λ_2, then their Bondi energy coincides on the interval $[\lambda_1, \lambda_2]$. Obviously, the Bondi energy of any static spacetime is constant and equal to its ADM energy.

As we claimed in the introduction, these general results on Bondi energy and its relation to ADM energy bridge between our findings on RG flows and the general expectations for time evolution in string theory. Below we will study RG flows from UV to IR fixed points on the world-sheet of the string; we will demonstrate that whenever quantities in the UV and IR limits can be well approximated by gravity computations, ADM energy of the UV fixed point is larger than the ADM energy of the IR fixed point. On the other hand, if we take the background fields at the UV fixed point in a region of light-cone time $\lambda < \lambda_1$, perturb them slightly at $\lambda \sim \lambda_1$ and let the perturbed system evolve in time using the gravity equations of motion, the system is expected to settle down into a stable static solution plus radiation at some later light-cone time $\lambda_2 > \lambda_1$. The Bondi energy of the resulting spacetime is $E^{(1)}_{\text{ADM}} = E^{\text{UV}}_{\text{ADM}}$ for $\lambda < \lambda_1$ and it is equal to $E^{(2)}_{\text{ADM}} = E^{\text{IR}}_{\text{ADM}}$ at $\lambda > \lambda_2$. Since Bondi energy is known to decrease we conclude $E^{(1)}_{\text{ADM}} \geq E^{(2)}_{\text{ADM}}$, just as for our RG flows on the world-sheet.

2.2 The Zamolodchikov c-theorem

The purpose of this subsection is to review the Zamolodchikov c-theorem. Our discussion follows the presentation and notation of [25].

In a rotationally invariant euclidean two-dimensional quantum field theory, two-point functions of stress energy tensors are constrained to take the following form

$$\begin{aligned}
\langle T_{zz}(z)T_{zz}(0)\rangle &= \frac{F(r)}{z^4}\,, \\
\langle T_{z\bar{z}}(z)T_{zz}(0)\rangle &= \frac{G(r)}{4z^3\bar{z}}\,, \\
\langle T_{z\bar{z}}(z)T_{z\bar{z}}(0)\rangle &= \frac{H(r)}{16z^2\bar{z}^2}\,,
\end{aligned} \tag{2.2}$$

where $r = \sqrt{z\bar{z}}$. The numerical factors on the right hand side have been chosen for later convenience. Ward identities associated with conservation of the stress tensor yield

$$\begin{aligned} \bar{\partial}\langle T_{zz}(z)T_{zz}(0)\rangle + \partial\langle T_{z\bar{z}}(z)T_{zz}(0)\rangle &= 0\,, \\ \bar{\partial}\langle T_{zz}(z)T_{z\bar{z}}(0)\rangle + \partial\langle T_{z\bar{z}}(z)T_{z\bar{z}}(0)\rangle &= 0\,. \end{aligned} \tag{2.3}$$

Combining (2.3) and (2.2) we find

$$\begin{aligned} 4\dot{F} + \dot{G} - 3G &= 0\,, \\ 4\dot{G} - 4G + \dot{H} - 2H &= 0\,. \end{aligned} \tag{2.4}$$

where $\dot{a} = r(da/dr)$. Defining

$$C(r) = 2F(r) - G(r) - \frac{3H(r)}{8}\,, \tag{2.5}$$

we have

$$\dot{C} = -\frac{3}{4}H\,. \tag{2.6}$$

At a conformal point G and H vanish (since $T_{z\bar{z}}$ vanishes in any conformal field theory) while $2F$ reduces to c, the constant central charge. Consequently $C(r) = c$ in a conformal field theory. Away from conformal points, $C(r)$ is generically not constant. In fact, the positivity of $H(z)$ in a unitary field theory implies that $C(r)$ is a monotonically non-increasing function of radius. Equivalently, by the Callan-Symanzik equation, $C(r)$ is a pointwise non-increasing function along renormalization group flows[3]. As we have already seen it also reduces to the constant central charge at conformal points. These statements are the content of the Zamolodchikov c-theorem.

To illustrate some special issues in sigma-models with non-compact target space it is useful to recast equation (2.6) by inserting the well known equation for the trace anomaly in a quantum field theory,

$$T_{z\bar{z}} = -\pi\beta^a O_a\,. \tag{2.7}$$

Here the O_a form a basis of relevant and marginal operators. We find that the C function

$$\begin{aligned} \dot{C} &= -12\pi^2 z^2\bar{z}^2\beta^a\beta^b\langle O_a(z)O_b(0)\rangle \\ &= -12\pi^2\beta^a\beta^b\mathcal{G}_{ab}(z) \end{aligned} \tag{2.8}$$

[3]This seemingly innocuous claim requires several qualifications. As was emphasized in [26] (see also [27, 28]), the action for a two dimensional field theory on a flat world sheet does not uniquely specify the stress tensor for the theory; the ambiguity in the stress tensor corresponds to the freedom in the choice of the dilaton. $C(r)$ decreases pointwise under RG flows only if the ambiguity in the definition of the stress tensor is fixed, scale by scale, to insure that $T_{\mu\nu}$ obeys the naive Callan-Symanzik equation. Such a redefinition of the stress tensor is always possible (see [26]); in fact the redefined stress tensor, as a function of scale may be determined by the requirement that the dilaton evolve according to its RG equations. This will be the prescription employed in the next section for the computation of the C function in perturbation theory. The application of the Callan-Symanzik equation to $C(r)$ is also complicated by the usual infrared divergences associated with massless two dimensional scalars [29] on attempting to expand them about a point in their moduli space. Presumably these divergences may be dealt with in the usual fashion (shifting to the correct vacuum, a wave function on moduli space) and do not present a problem in principle. We would like to thank H. Osborn, A. Strominger, A. Tseytlin and especially J. Polchinski for very illuminating discussions on this topic.

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can be expressed through the Zamolodchikov metric,

$$\mathcal{G}_{ab}(z) = z^2 \bar{z}^2 \langle O_a(z) O_b(0) \rangle \,. \tag{2.9}$$

and hence through the matrix of two point functions. The latter is schematically given by

$$\langle O_a(0) O_b(z) \rangle = \frac{\int \mathcal{D}X^i e^{-S[X]} O_a(X(0)) O_b(X(z))}{\int \mathcal{D}X^i e^{-S[X]}} \,. \tag{2.10}$$

As observed in [13, 30], the denominator of the matrix element (2.10) is proportional to the volume of the target space (from the integral over the zero mode) and hence it is infinite for our non-compact backgrounds. We will cut off this integral at V_0 by imposing an IR regulator (compactification) on the target space. But we also assumed that our RG flows are generated by relevant operators which are localized in target space. Hence, the integral over the zero mode in the numerator of the matrix (2.10) converges. We conclude that the matrix elements $\mathcal{G}_{ab}$ are of order $1/V_0$, and therefore the C function $C(r)$ changes along the RG flow only at this sub-leading order. If we remove the cutoff V_0 then $C(r)$ remains constant—as anticipated in the introduction. But we will find that the first correction to this constant term in $C(r)$ contains precisely the information we are looking for, i.e. that it can be used to determine the direction of an RG flow.

2.3 The C function in sigma model perturbation theory

In this subsection we present some results from Tseytlin's perturbative calculation of the function $C(r)$ (defined in eqs. (2.2) and (2.5)) to first order α'. We will utilize these results in the next section, to demonstrate that spacetime energy decreases along world-sheet RG flow.

Consider the type-II sigma model defined by the action[4]

$$S^{(2)} = \frac{1}{2\pi\alpha'} \int d^2z \left(\left(G_{ij}(X) + B_{ij}(X)\right) \partial X^i \bar{\partial} X^j + \frac{\alpha'}{4} \left(\Phi(X) + \Phi_0\right) R^{(2)} + \text{fermions} \right) . \tag{2.11}$$

If the model is not conformal, then the spacetime fields G_{ij}, B_{ij}, Φ will depend on the renormalization scale k. Tseytlin [22] has computed Zamolodchikov's function $C(r)$ for this model at first order in the α' expansion. His results involve a 'running central charge' $s(k)$, whose scale dependence is converted into the r-dependence of $C(r)$ by a generalized Fourier transform.

To spell out the details we first define $s(k)$. Let us recall that the beta functions for the dilaton and metric in sigma model possess an expansion of the form

$$\begin{aligned} \beta_{ij} &= \alpha' \left(R_{ij} + 2\nabla_i \nabla_j \Phi - \frac{1}{4} H_{ikl} H_j{}^{kl} \right) + \mathcal{O}\left(\alpha'^2\right) , \\ \beta^{\Phi} &= \alpha' \left(-\frac{1}{2} \nabla^2 \Phi + (\nabla \Phi)^2 - \frac{1}{24} H^2 \right) + \mathcal{O}\left(\alpha'^2\right) . \end{aligned} \tag{2.12}$$

[4]It will be convenient for us to work with a shifted dilaton Φ which vanishes at infinity. Correspondingly, κ is the asymptotic value of the gravitational coupling, proportional to e^{Φ_0}. In addition we will set all R-R fields to zero.

Here we have included in β^Φ the ghost contribution that cancels the zeroth-order term $d/4$. The spacetime action can be written in terms of these beta functions:

$$\begin{aligned} S(k) &= \frac{1}{2\kappa^2\alpha'}\int d^dx\sqrt{G}e^{-2\Phi}\left(G^{ij}\beta_{ij} - 4\beta^\Phi\right) \\ &\sim \frac{1}{2\kappa^2}\int d^dx\sqrt{G}e^{-2\Phi}\left(R^{(d)} - \frac{1}{12}H^2 - 4(\nabla\Phi)^2 + 4\nabla^2\Phi\right). \end{aligned} \tag{2.13}$$

From the first line we infer that the spacetime lagrangian is a linear combination of beta functions and hence it vanishes on shell.

Next we define

$$V(k) = \frac{1}{2\kappa^2}\int d^dx\sqrt{G}e^{-2\Phi}\,, \tag{2.14}$$

which—by slight abuse of terminology—we will refer to as the "volume" of space. As in the previous section, we shall give meaning to the divergent quantity $V(k)$ by imposing a target space IR cutoff V_0. Note that, by (2.12),

$$kV'(k) = \frac{\alpha'}{2}S(k)\,. \tag{2.15}$$

This relation will be useful below in estimating the change in the volume during the flow.

Tseytlin's 'running central charge' $s(k)$ is defined in terms of the spacetime effective action and the volume of spacetime by

$$s(k) = \frac{3d}{2} - \frac{3\alpha' S(k)}{2V(k)}\,. \tag{2.16}$$

Note that $s(k)$ is designed to equal the central charge at conformal points (recall that S vanishes on shell). The Zamolodchikov C function can be obtained from $s(k)$ by a generalized Fourier transform,

$$C(r) = \int_0^\infty \frac{dk}{k} f(kr)s(k)\,, \tag{2.17}$$

whose kernel is given by a combination of Bessel functions:

$$f(x) = -\frac{1}{2}x^2 J_0(x) + \frac{1}{4}x^3 J_0'(x) - xJ_0'(x)\,. \tag{2.18}$$

(it is understood that the integral in eq. (2.17) is computed with a damping factor $e^{-\epsilon k}$, where ϵ is taken to zero at the end of the computation). Equations (2.17), (2.13), (2.14), and (2.18) together constitute Tseytlin's result for $C(r)$ to first order in α' perturbation theory.

The Fourier kernel function (2.18) is rather complicated, but only two simple properties will be of importance to us. Firstly, we have the following normalization condition,

$$\int_0^\infty \frac{dx}{x} f(x) = 1\,, \tag{2.19}$$

which may be verified either by direct computation or by applying (2.17) to a conformal sigma model in which both $C(r)$ and $s(k)$ are constant and equal to the central charge. Secondly, we have $f(x) \sim x^4$ as $x \to 0$.

While the detailed expressions we have presented above apply only to first order in the α' expansion, the equations of this subsection may formally be generalized to exact relations. In particular (2.16), (2.17), and (2.18) actually hold as exact relations when the quantities $S(k)$ and $V(k)$ are defined by slight generalizations of the first equation in (2.13), and equation (2.15). We will not present the detailed expressions here, but merely note that the exact expression for the lagrangian in $S(k)$ turns out always to be proportional to a linear combination of β functions, and so vanishes on shell. This fact will be important for us in section 3 below.

3. Target space energy decreases under world-sheet RG flow

We now turn to the study of a class of RG flows in non-compact unitary sigma models. Specifically, we will combine the explicit computation of $C(r)$ in α' perturbation theory with the Zamolodchikov c-theorem. We shall thereby be able to obtain all the results we listed in the introduction.

To begin, let us be precise about the assumptions we make. We study RG flows with the following properties:

1. The UV and IR sigma models have the same asymptotic behavior at spatial infinity. In two spatial dimensions we are able to relax this assumption and to allow for different asymptotic conical deficit angles in the UV and the IR.

2. The RG flow is seeded by a localized tachyon, and proceeds by an expanding bubble of the final solution (i.e. the IR background) embedded in the initial solution (i.e. the UV background) with some transition region connecting them (see figure 2).[5]

3. All length scales in the asymptotic region of the geometry are large in string units. This restriction is imposed to permit the use of sigma model perturbation theory in the asymptotic region. Note that we do not require sigma model perturbation theory to apply uniformly on target space: flows involving orbifold fixed points or other singularities are perfectly acceptable. Technically, we will apply the α' expansion only in the transition region of figure 2, and only after RG flow has proceeded sufficiently far so that the transition region is well within the asymptotic regime.

3.1 General strategy

The main aim of this subsection is to outline the strategy we shall follow in order to establish the claims we made in the introduction. Some of the necessary but more technical computations will be deferred to later subsections. Our argument essentially proceeds in two steps. In the first step we show that $S(k)$ tends to constant for $k \ll k_1$ where k_1 is the RG scale at which the expanding transition region lies entirely in the asymptotically

[5]Adams, Polchinski, and Silverstein [10] first explained that tachyon condensation proceeds in this fashion in certain models. See appendix A for an exact solution of the supergravity RG flow that illustrates this bubble nucleation and growth in the simple context of two dimensional sigma models.

flat region of spacetime. Furthermore, we compute the value of $S(k)$ in this infrared region using results from subsection 2.3 and find

$$S(k) \sim -\Delta E_{\rm ADM} \equiv E_{\rm ADM}^{\rm IR} - E_{\rm ADM}^{\rm UV}\,, \qquad k \ll k_1\,, \tag{3.1}$$

where the ADM energies for the IR and UV fixed points are computed from the Einstein frame background fields using (2.1). Equation (3.1) is established in subsection 3.2–3.4.

Once we know the behavior (3.1) of $S(k)$ in the infrared, we can pass to the second step. With the help of formulae (2.17), (2.19) we will be able to conclude

$$\Delta E_{\rm ADM} = \lim_{V_0\to\infty} \frac{2V_0}{3\alpha'}\Delta C\,. \tag{3.2}$$

Here ΔC represents the difference between the C function at very large and very small values of r. The formula (3.2) is the principal result of this paper. According to the c-theorem, the right hand side of (3.2) is negative. Hence we conclude that $\Delta E_{\rm ADM}$ is negative, and ADM energy decreases under RG flow.

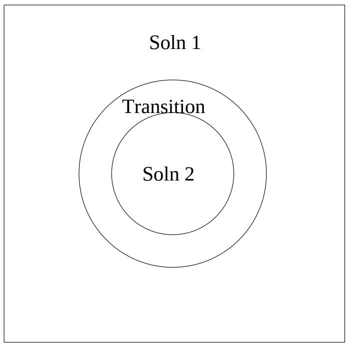

Figure 2: As RG flow proceeds, a bubble of the final solution ("Soln 2") is nucleated and grows. This bubble is connected to the surrounding sea of the initial solution ("Soln 1") by an interpolating transition region.

In the remaining part of this subsection we would like to prove eq. (3.2), assuming (3.1) (which will be proved in later subsections). We must first admit, however, that in the paragraphs above we have glossed over some important subtleties which arise from the IR (in target space) cutoff V_0. The latter determines the volume of the initial (UV) target space, i.e. $V_0 = V(k{=}\infty)$. Since this cutoff is merely an artificial device, we are not interested in the details of the flow once the bubble of figure 2 becomes so large that it begins to sense the IR cutoff.[6] Since the RG flow equations are diffusive in character, we can estimate that this occurs at an RG scale k_0 determined by

$$-\ln(\alpha' k_0^2) \sim \frac{V_0^{2/d}}{\alpha'}\,. \tag{3.3}$$

Indeed, the result (3.1) that we quoted above is strictly valid only for $k \gg k_0$, and may more accurately be stated as

$$S(k) \sim -\Delta E_{\rm ADM}\,, \qquad k_0 \ll k \ll k_1\,, \tag{3.4}$$

where, as before, k_1 is the finite RG scale at which the transition region in figure 2 first lies entirely in the asymptotic region of target space geometry.

With these remarks in hand we are prepared to prove eq. (3.2). The idea is to insert eq. (3.4) into eq. (2.17) and thereby relate the behavior of $S(k)$ in the infrared to the C function. At first sight it seems a bit problematic that we do not know the value of $s(k)$ for $k < k_0$ or $k > k_1$. However the c-theorem and unitarity ensure that it is of unit order. Consequently, the contribution to $C(r)$ from the region $0 \le k \le k_0$ in the

[6] In particular, this allows us to neglect any boundary terms that might arise due to the IR cutoff, since they will not change during the part of the flow that interests us.

integral (2.17) is of order $r^4 k_0^4$ (recall that $f(x) \sim x^4$ at small x). For $r \sim {\alpha'}^{1/4} k_0^{-1/2}$, this contribution scales like $\exp(-2V^{2/d}/\alpha')$, and so is negligible compared to the dominant effects of order $1/V_0$. Similarly, the contribution of $k > k_1$ to the integral (2.17) is of order $(\int_{k_1 r}^{\infty} dx f(x)/x)/V_0$, and so is negligible compared to the dominant effect (recall that, from (2.19), $\lim_{r\to\infty} \int_{k_1 r}^{\infty} dx f(x)/x = 0$).

Based on these estimates it is rather easy to establish (3.2), or rather the following more accurate version:

$$\Delta E_{\mathrm{ADM}} = \lim_{V_0\to\infty} \frac{2V_0}{3\alpha'} \left(C({\alpha'}^{1/4} k_0^{-1/2}) - C(0) \right). \tag{3.5}$$

Note that the limit on the right hand side is well defined and finite. In fact, according to our discussion in subsection 2.2, the C function possesses a universal term that is independent of the cutoff and a more interesting subleading contribution at order $1/V_0$. The first term is removed when we substract the value of the C function at two different arguments and hence the leading non-zero contribution to the difference appears at order $1/V_0$. Its coefficient is extracted by multiplying with V_0 and sending V_0 to infinity. This is exactly what we are instructed to do on the right hand side of eq. (3.5).

It may have puzzled the reader that we treated the running volume $V(k)$ as if it was a constant. This is actually justified because (as may be seen using eqs. (2.15) and (3.3)) the fractional change of $V(k)$ between the UV and the scale k_0 goes to zero in the limit of large V_0.[7]

It remains to prove relation (3.4). As we have stressed above, the spacetime action $S(k)$ receives no contributions from regions in which the field configurations are on shell. Consequently, the action of a field configuration of the form depicted in figure 2 receives contributions only from the transition region. We now proceed to verify eq. (3.4) by evaluating the action of this transition region for three cases in turn, beginning with the most straightforward one, namely asymptotically flat backgrounds in $d > 2$.

3.2 Asymptotically flat backgrounds

Here we assume that both the initial and final target spaces are asymptotically flat solutions to the Einstein equations. By the Newton law, all fields in these solutions deviate at a distance R from their flat space values at order R^{2-d}. Hence if we let the RG flow proceed until the bubble of final solution ("Soln 2" in figure 2) is of radius R, then field fluctuations in the interpolating transition shell are also of order R^{2-d}. Thus for $d > 2$ and at large R all fields in the transition shell are very near their flat space values, and so the action of the transition band is well approximated by expanding the action in powers of the fluctuations of fields. In fact, as $R \to \infty$, this expansion need only be performed to linear order; quadratic and higher terms in the expansion are negligible. For example, the standard

[7]This statement actually is only true for $d > 2$. For $d = 2$ the same fractional change goes to a constant at the scale k_0. This can be derived by considering two truncated cones with the same base circumference but different deficit angles. At larger scales such as $k_0^{3/4}$, however, the fractional change is zero and this is sufficient for the purposes of our argument.

quadratic kinetic term for the dilaton, evaluated on the transition region, is

$$\int d^d x \, (\partial\Phi)^2 = \mathcal{O}\left(R^{2-d}\right), \tag{3.6}$$

and it may therefore be dropped in the limit $R \to \infty$. Higher order terms and terms with more derivatives are further suppressed. Indeed the only terms that survive this limit are two-derivative terms linear in fluctuations (e.g. $\int d^d x \, \partial^2 \Phi$) which are clearly of order one.

Thus we need only retain the following terms from the action (2.13):

$$S_{\text{trans}} = \frac{1}{2\kappa^2} \int_{\text{trans}} d^d x \sqrt{G} e^{-2\Phi} \left(R^{(d)} + 4\partial^2 \Phi\right). \tag{3.7}$$

Since the ADM energy is defined in terms of the $d+1$ dimensional Einstein frame metric,

$$g_{00} = -e^{-4\Phi/(d-1)}, \qquad g_{ij} = e^{-4\Phi/(d-1)} G_{ij}, \qquad g_{0i} = 0, \tag{3.8}$$

we prefer to express eq. (3.7) in terms of Einstein frame background fields. After dropping quadratic terms the result is

$$S_{\text{trans}} = \frac{1}{2\kappa^2} \int_{\text{trans}} d^d x \sqrt{-g^{(d+1)}} R^{(d)}_{\text{E}}, \tag{3.9}$$

where $g^{(d+1)}$ is the determinant of the $d+1$ dimensional metric (3.8), and $R^{(d)}_{\text{E}}$ is the Ricci scalar formed its spatial part g_{ij}. We can now expand to first order in fluctuations about flat space to obtain

$$S_{\text{trans}} = \frac{1}{2\kappa^2} \int_{\text{trans}} d^d x \left(\partial_i \partial_j h_{ij} - \partial^2 h_{ii}\right) = \frac{1}{2\kappa^2} \int_{\partial(\text{trans})} dS^j \left(\partial_i h_{ij} - \partial_j h_{ii}\right) = -\Delta E_{\text{ADM}}. \tag{3.10}$$

The minus sign in the last equality arises because the unit normal vector in the integration over the boundary of the transition region is directed to the outside of this region. In relating the integrals to the definition of the ADM energy we therefore have to invert the orientation on the inner shell that bounds the bubble of the final solution.

3.3 Arbitrary asymptotics

Let us generalize the result of the previous subsection to RG flows in which the asymptotic boundary conditions shared by the initial and final solutions may be arbitrary. In such a situation, while an absolute notion of total energy does not exist, the difference in energies ΔE_{ADM} continues to be well defined. For this purpose, we denote the difference between the two metrics as h_{ij},

$$g^{(\text{f})}_{ij} = g^{(\text{i})}_{ij} + h_{ij}, \tag{3.11}$$

and use the initial metric $g^{(\text{i})}_{ij}$ to raise and lower indices and calculate connection coefficients. The difference in energies between the two solutions is then [31]

$$\Delta E_{\text{ADM}} = \frac{1}{2\kappa^2} \int \sqrt{-g_{00}} (D_i h^{ij} - D^j h) dS_j . \tag{3.12}$$

As usual, the integral in (3.12) is taken over a surface at the boundary of space at a fixed time, and S_j is an outward-directed unit normal to the surface. The formula (3.12) holds for cases in which the metrics are independent of time, and the space-time metric components g_{0i} vanish.

We will now show that the action $S(k)$ in the deep IR equals precisely minus the change (3.12) in the ADM energy. As in the previous subsection this action receives contributions only from the transition shell, and it may be evaluated by an expansion to first order in fluctuations about the background. It is again useful to rewrite the action as a function of the $d+1$ dimensional Einstein frame metric (3.8):

$$S_{\mathrm{trans}} = \frac{1}{2\kappa^2}\int_{\mathrm{trans}} d^d x\sqrt{-g^{(d+1)}}\left(R_{\mathrm{E}}^{(d)} + \frac{1}{12}e^{-8\Phi/(d-1)}H^2 - \frac{4}{d-1}(\partial\Phi)^2\right). \tag{3.13}$$

Expanding the action to first order in fluctuations about the background we find

$$S_{\mathrm{trans}} = \frac{1}{2\kappa^2}\int_{\mathrm{trans}} d^d x\sqrt{-g^{(d+1)}}g^{ij}\mathrm{R}_{\mathrm{E}ij}^{(d)} + \cdots, \tag{3.14}$$

where $\cdots$ refers to terms that vanish by the equations of motion on the background field. Utilizing the Palatini formula for the variation of the Ricci tensor we find

$$\begin{aligned} S_{\mathrm{trans}} &= \frac{1}{2\kappa^2}\int_{\mathrm{trans}} d^d x\sqrt{-g^{(d+1)}}\left(D_i D_j h^{ij} - D^2 h\right) \\ &= \frac{1}{2\kappa^2}\int_{\partial(\mathrm{trans})}\sqrt{-g_{00}}(D_i h^i{}_j - D_j h^i{}_i)dS^j \\ &= -\Delta E_{\mathrm{ADM}}. \end{aligned} \tag{3.15}$$

3.4 Analysis for $d = 2$

In 2+1 dimensions, solutions to the Einstein equations that asymptotically take the form

$$ds^2 = -dt^2 + dr^2 + \zeta^2 r^2 d\phi^2 \tag{3.16}$$

may be assigned an ADM energy proportional to their deficit angle:

$$E_{\mathrm{ADM}} = \frac{2\pi}{\kappa^2}(1-\zeta)\,. \tag{3.17}$$

Here we shall consider RG flows that interpolate between two solutions of the form (3.16) where the UV deficit angle is ζ_1 and the IR deficit angle is ζ_2.[8] As H_{ijk} is always zero in two dimensions, the string effective action (2.13) takes an extremely simple form for such flows

$$S = \frac{1}{2\kappa^2}\int d^2 x\sqrt{-G}\,R\,. \tag{3.18}$$

The integrand on the right hand side is a total derivative. When integrated over the transition region it evaluates to

$$S_{\mathrm{trans}} = \tfrac{2\pi}{\kappa^2}(\zeta_2 - \zeta_1) = -\Delta E_{\mathrm{ADM}}\,. \tag{3.19}$$

[8]In dimensions $d > 2$ we studied transitions between asymptotically flat backgrounds with nontrivial dilaton tails $\Phi \sim 1/r^{d-2}$. For $d = 2$ such a dilaton tail $\Phi \sim \ln(r)$ would destabilize the background (3.16). Consequently, we restrict attention to asymptotically constant dilaton profiles.

Our ability to control flows that change the asymptotic geometry in two but not in higher dimensions may be attributed to the fact that solutions with distinct deficit angles in two dimensions differ in energy by a finite amount.

Appendix A provides an illustration of such flows by exhibiting an exact solution to the RG flow equations at first order in α' that interpolates between spaces with arbitrary ζ_1 and $\zeta_2 < \zeta_1$.

4. Discussion

In this note we have studied renormalization group flows that interpolate between two spacetime solutions with a finite energy difference. These processes include flows on non-compact sigma models with fixed asymptotics in $d > 2$, and flows that interpolate between solutions with different asymptotic deficit angle in $d = 2$. In all these examples we have argued that RG flow proceeds in the direction that reduces spacetime energy.

In our proof we have assumed that it is possible to impose an infrared cutoff on the target space in order to make CFT_2 compact. While we do not have a general proof that such a cutoff always exists, we can give examples in simple cases. C/Z_n orbifolds can be embedded in compact T^2/Z_n CFTs for $n = 2, 3, 4, 6$ [11]. For other values of n, a straightforward compactification can be achieved by adding an extra dimension: consider a level k SU(2) WZW model orbifolded by a Z_n subgroup of SU(2). For large k, in the vicinity of the orbifold fixed line, the target space will approximate $R \times C/Z_n$. The size of the regulator here is controlled by k. It would be interesting—and useful—to find such regulators for more complicated unstable backgrounds.

It would also be interesting to check explicitly the validity of the c-theorem for the expressions derived in [22] for the C function.

Several recent examples of tachyon condensation (including the decay of C^2/Z_n orbifolds of type-II theory [10]–[13], the decay of twisted circle or Melvin compactifications of string theory [14, 32, 33], and the decay of non-abelian flux backgrounds in the heterotic theory [15]), involve more violent processes, namely RG flows that change the asymptotics. The analysis presented in this paper does not generalize directly to such flows. The spacetime action $S(k)$ does not tend to a constant value in the IR (as in eq. (3.1)) but instead increases without bound.[9] The c-theorem would appear to imply the positivity of $S(k)$ at small k, but it is not obvious how to translate this statement into a restriction on RG flows.

Although we are not able to make this conjecture precise, it is tempting to speculate that the violent RG flows discussed in the previous paragraph also proceed so as to reduce spacetime energy in some sense.[10] Indeed, the most satisfactory completion of the line of argumentation presented in this paper would be the identification of spacetime energy with a natural construction in two dimensional field theory, and a demonstration that this quan-

[9]For example, in flows that interpolate between C^2/Z_n and C^2/Z_m one finds that $S(k) \sim -\ln(k\sqrt{\alpha'})$ by power counting.

[10]See appendix B for some thoughts in this direction, and [12] and [13] for related proposals.

tity decreases along world-sheet RG flows.[11] Indeed, a solution to the equivalent problem for open strings was proposed long ago, and is by now well understood in the context of open string tachyon condensation [35]. The open string theory partition function on the disk (which is related to the boundary entropy in boundary conformal field theory [36]) may be identified with spacetime energy and appears to decrease along RG flows [37]. It is possible that a suitably regulated version of the closed string partition function on the sphere enters into a similar construction for closed strings[12] (see [38, 39] for some work in this direction). We postpone speculations in this direction to future work.

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A. Exact RG solution for the decay of a cone

The RG flow equation for a sigma model onto a purely gravitational target space is, to first order in α',

$$\frac{dG_{\mu\nu}}{d\lambda} = -\alpha' R_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \,, \tag{A.1}$$

where the vector ξ is arbitrary, representing the freedom to make continuous changes of target space coordinates along the flow. In [10] an approximate solution to this equation was presented describing the decay of a cone C/Z_n to another one $C/Z_{n'}$ ($n' < n$), for large n and n'. Here we shall present an exact solution describing the decay of a cone from any deficit angle to any smaller deficit angle.

For simplicity let us begin with the case where the cone, with initial deficit angle $2\pi(1-\zeta)$, decays to the plane. The solution is

$$ds^2 = \lambda\left(f^2 dr^2 + r^2 d\phi^2\right), \qquad \xi = \frac{1}{2} r f dr, \tag{A.2}$$

or, by making the λ-dependent change of coordinates $r = \rho/\sqrt{\lambda}$,

$$ds^2 = f^2 d\rho^2 + \rho^2 d\phi^2 \,, \qquad \xi = \frac{1}{2\lambda}\rho f(1-f) d\rho \,. \tag{A.3}$$

[11] A world-sheet definition of spacetime energy which is valid on shell has been formulated recently in [34].

[12] We thank E. Witten for this suggestion.

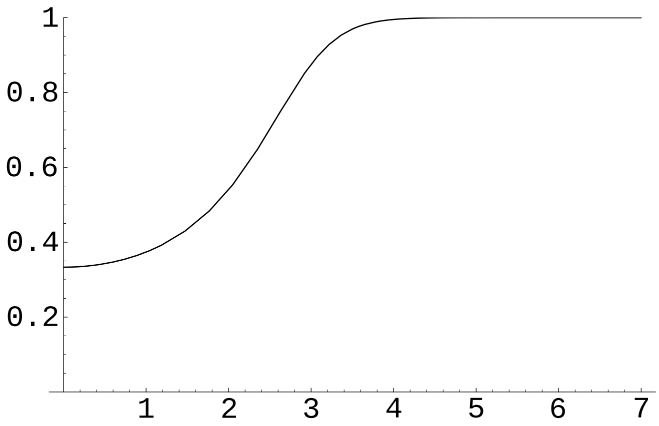

Figure 3: The function f defined in (A.4), plotted against $r/\sqrt{\alpha'}$, for the case $\zeta = 1/3$.

Here $r, \rho \in [0, \infty)$, $\phi \sim \phi + 2\pi\zeta$, and f is a function that interpolates smoothly between $f = \zeta$ at $r = 0$ and $f = 1$ at $r = \infty$ (see figure 3). Specifically,

$$f = \left[1 + W\left((\frac{1}{\zeta} - 1) \exp\left(\frac{1}{\zeta} - 1 - \frac{r^2}{2\alpha'} \right) \right) \right]^{-1}, \tag{A.4}$$

where W is the product log function, the inverse function of xe^x.

The RG flow parameter λ in this solution ranges only from 0 to ∞. The limits $\lambda \to 0$ and $\lambda \to \infty$ are best taken at fixed ρ (rather than fixed r), giving respectively the cone and the plane. For finite λ, as shown in figure 4, the geometry is conical at infinity but smooth at the origin, with typical radius of curvature $\sqrt{\lambda\alpha'}$ (to be precise, the Ricci scalar is $R = 2(f^{-1} - 1)/\lambda\alpha'$). Thus the curvature which was initially concentrated at the origin eventually diffuses over an infinite area. We can only trust this solution for $\lambda \gg 1$, when the curvature is small enough for eq. (A.1) to be valid. Hence there is no significance to the fact that we cannot continue this solution to negative λ, and as usual, we expect the full RG flow to extend from $\lambda = -\infty$ to $\lambda = \infty$.

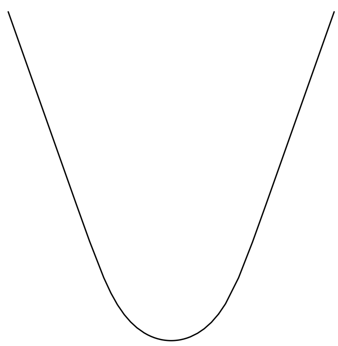

Figure 4: Cross section of the cone (A.2), (A.3) in the case $\zeta = 1/3$.

The first form (A.2) makes it manifest that the flow proceeds by a global Weyl transformation, i.e. the geometry keeps the same shape while expanding in size. This is analogous to the gaussian solution of a linear diffusion equation, which over time broadens but always remains a gaussian. In this sense a gaussian is a "fixed point modulo broadening" of the diffusion equation, and it is this property which implies that an arbitrary initial distribution (with a finite total amount of matter) will after a sufficiently long time diffuse into a gaussian distribution. Analogously, since the above solution is a "fixed point modulo global Weyl transformations" of eq. (A.1), we can expect that if at $\lambda \approx 1$ the geometry consists of a cone with a smoothed-out vertex of size $\sim \sqrt{\alpha'}$, then for large λ the flow will be better and better approximated by the solutions (A.2) and (A.3), with errors of order $1/\lambda$. This is important because it implies that, in studying the RG flow of a C/Z_n orbifold, it is not necessary to know the precise configuration at the end of the string-scale regime in order to know how the flow will proceed in the subsequent supergravity regime.

The solution can easily be generalized to describe the decay of a cone with $\zeta = \zeta_1$ to one with $\zeta = \zeta_2 > \zeta_1$. Simply let the periodicity of ϕ be $2\pi\zeta_1$, and set $\zeta = \zeta_1/\zeta_2$ in the expression (A.4) for f.

B. Speculations on asymptotics-changing RG flows

In this paper we have argued that spacetime energy decreases along RG flows for which the asymptotic geometry of spacetime remains constant along flows. Though it is tempting to speculate that this result applies to more general RG flows, it is difficult to formulate a precise conjecture along these lines, as it is unclear how the energies of solutions with different asymptotics may be compared.

In this connection, however, we find it intriguing to note that Hawking and Horowitz have derived a rather beautiful universal and geometrical formula [31]

$$E_{\rm HH} = -\frac{1}{8\pi}\int \sqrt{-g_{00}}K \tag{B.1}$$

for the energy of spacetimes of the form

$$ds^2 = g_{00}dt^2 + g_{ij}dx^i dx^j \tag{B.2}$$

where the integral is taken over the boundary of space at a fixed time, and K is the extrinsic curvature of this boundary viewed as a sub-manifold of a constant time slice of the metric (B.2). Unfortunately, the relationship (B.1) is rather formal since the right hand side diverges for most spacetimes. To arrive at a finite quantity one again has to subtract the contribution of a reference background in (B.1).

Nonetheless eq. (B.1) is useful and appears to fit well with world-sheet RG flows. It was used in [31] to derive the formula (3.12) for the difference in energy between two nearby spacetimes. It is conceivable that eq. (B.1) may also be employed, together with a regulation prescription motivated by a consideration of RG flows, to determine the sign of the (generically divergent) energy differences between spacetimes of different asymptotics.

To illustrate the idea, consider an RG flow from a C^2/Z_n orbifold to a C^2/Z_m orbifold of type-II theory. The extrinsic curvature of a sphere of radius R is $1/R$. Consequently, the energy (B.1) of the C^2/Z_n, computed on a sphere of radius R_1, is $-R_1^2/8\pi n$. Similarly the energy of the C^2/Z_m orbifold computed on a sphere of radius R_2 is $-R_2^2/8\pi m$. As a toy model of an RG flow motivated regulation procedure, we prescribe that the energies of C^2/Z_n and C^2/Z^m should be compared on surfaces of equal surface area, i.e. when $R_1^3/n = R_2^3/m$. With this prescription,

$$\Delta E = \frac{R_1^2}{8\pi n}\left(1 - \left(\frac{n}{m}\right)^{1/3}\right). \tag{B.3}$$

If we conjecture that energy decreases along RG flows then we derive a constraint on these flows: The order of the orbifold must decrease in the process of tachyon condensation.

This prediction happens to be fulfilled in all examples of such RG flows that we are aware of. Nonetheless the analysis of this paragraph should not be taken too seriously. It is presented merely as a cartoon of an detailed analysis which remains to be worked out and may succeed in using an energy function like (B.3) to constrain RG flows.

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